ABSTRACT. In this paper I analyze the forest and debt dynamics in a less developed country (LDC), where the former is a renewable resource and the latter’s increase results from the interests to be paid on the current debt minus the balance of trade surplus. Agricultural and industrial goods are produced, and whereas the former requires the converted forest as an input, the latter does not. It transpires that the stock of debt is likely to increase infinitely without repudiation, whereas the stock of forest is likely to oscillate around an equilibrium level. Within this framework, I compare the effectiveness and enforceability of the debt-for-nature and the debt-for-development swaps with respect to tropical deforestation and debt burden issues. Some empirical evidence confirming the theoretical results is provided.

Resource management in less developed countries (LDCs) is likely to have a substantial impact on the world environment over the long term: this has led to a widespread concern about desertification, deforestation, and biodiversity reduction issues. The evaluation of environmental improvement in LDCs, however, may be much less than in developed countries (DCs) (Dasgupta, 1993): this has raised the issue of implementing effective initiatives by DCs for the preservation of LDCs resources.

LDCs have faced drastically increasing debt burdens over recent years due to dramatic increases in real interest rates, deteriorating terms of trade, and poor returns on money borrowed. The persistence of these three phenomena has contributed to the development of a broad menu of environmental management and debt adjustment arrangements such as the so-called debt-for-nature swap (DFNS). Two kinds of DFNS have been agreed upon:

1. A typical private swap involves at least three parties: (i) an international conservation organization that often initiates the formal process and
purchases part of the LDC’s debt at a discounted value in the secondary market. The three most active have been Conservation International, the Nature Conservancy, and the World Wildlife Fund; (ii) a national conservation organization in the host country that is linked to the international conservation organization by a working relationship, and that implements the environmental actions. These are predominantly non-government organizations; (iii) one or more government agencies from the host country. These includes the following two representations: the central bank that agrees to convert a portion of its external debt to domestic currency obligations and that specifies an upper limit to conversions, an applicable exchange rate, and a conversion rate; and an agency that receives and channels the international assistance and has general oversight responsibilities.

2. In a typical public swap the international conservation organization is replaced by a DC government. Notice that at least one party of such a transaction is a sovereign nation.

So far the DFNSs have mobilized only a small proportion of the resources needed to support environmental sustainability and it has influenced only a small portion of the outstanding debt. Since 1987 16 countries have converted $159 million in face value of debt through such programs. Moreover, the DFNSs have turned out only to be appropriate for some countries and its importance has been declining over time (World Bank, 1996a).

A more recent debt adjustment arrangement is the so-called debt-for-development swap (DFDS).

A typical DFDS shows the same structure of a DFNS except for the international organizations involved and the projects implemented. In particular, the three main participants in DFDSs have been the Finance for Development Fund, New York Bay Fund, and the United Nations Children Fund. The funded projects have been in sectors such as health, population, agriculture, ecotourism, low-income housing, primary education, women in development, water supply, and sanitation.

Notice that again at least one party to such a transaction is a sovereign nation.

So far the DFDSs have mobilized a larger portion of the outstanding debt than the DFNSs. By 1996 UNICEF had completed 22 transactions reducing the outstanding debt by $199 million, whereas the Finance for Development Fund and New York Bay Fund have swapped $391 million since 1992. Moreover, additional debt conversion operations are planned (World Bank, 1997).

The present paper compares the effectiveness and the enforceability of the DFNS and the DFDS with respect to tropical deforestation and debt burden issues.

As regards their effectiveness, I consider to what extent these swaps affect the stock of forest and the stock of debt in the short run as well as in the long run.

As regards their enforceability, it is clear from the description above
that these swaps imply the following contractual problems: (i) they involve transaction costs and impose risks on the parties involved; (ii) they include commodities such as resource conservation and development that are vague and subject to interpretation; (iii) they involve actions difficult to monitor and control; (iv) they include a sovereign nation as one party.

There is a vast literature focused on how a DFNS deals with these problems.

Some authors analyze the structure that this contract has taken over time (Curtis and Tourreilles, 1992; Fuller, 1989; Hansen, 1989; Patterson, 1990; Reilly, 1990; Wagner, 1990). In particular, they show that both private and public DFNSs have evolved in three main ways: (i) the host country government is less frequently included as an active party; (ii) the activities to be undertaken and the funds to be applied, instead of the objectives to be obtained, are more often described in the project proposals; (iii) the conservation organization more often avoids direct ownership by itself or other foreign agencies because of negative population reaction to foreign ownership of a national park or reserve; (iv) funds are less frequently assigned to designate land as government reserve, and to augment enforcement over established nominal title instead. However, the lack of a credible third-party enforcement for private swaps and their greater frequency relative to the public swaps highlights the factors that determine the consistency of the project proposals with the LDC self-interest.

Other authors have highlighted the conditions under which this swap may be self-enforcing (Chambers et al., 1996). In particular, within a static context, they show that complementary preservation projects are a necessary but not a sufficient condition for a DFNS to arise as a non-cooperative equilibrium.

However, it is well known that it is the outstanding stock of debt that worries LDCs as well as the scarcity of the stock of forest that bothers DCs. Hence a dynamic framework must be developed in order to take such features into account.

In this paper I consider which conditions may lead the LDC to implement over time the actions implied by the DFNS’s clauses.

Section 1 sets up a stylized model which describes the LDC economy. This economy contains a given amount of land split into forest and cleared land. The forest is a renewable resource so that its regeneration function depends upon the size of its current stock; the complementary cleared land dynamics is deduced. Since deforestation for agriculture is much more significant (Hyde et al., 1991), only the former process is assumed to affect the cleared land dynamics.

The economy produces two final products: one that requires the converted forest as an input and one that does not. These goods are called agricultural and industrial goods. This distinction captures the shift of the economy’s mix of final products.

Since the industrial good is obtained by using labor only, competitive firms in this sector simply maximize their current profit by choosing equilibrium labor.
Overexploitation of natural resources can arise for two main reasons: the structure of access to forest and the burden of the external debt.

Bromley (1991) shows that for cases where the access to the forest is common property, and Ciriacy-Wantrup and Bishop (1975) show that, for the cases where the access is completely open, there are no enforceable rights to any benefit stream generated by the forest. This means that individuals do not incorporate its future state into their decisions and consequentially overuse arises. On the other hand, Kahn and McDonald (1995) find evidence of a positive relationship between the levels of external debt and deforestation, whereas Murphy (1994) shows that this relationship is relevant only when countries face a binding credit constraint on their external borrowing. In a two-period framework, Strand (1995) analyzes the relationship between natural resource extraction, borrowing, and debt for a LDC that possesses a natural resource and studies the effect of policies by an outside donor on its rate of extraction.

In this paper I will consider the latter reason only. Thus, since the agricultural good is obtained by applying both labor and cleared land, competitive firms in this sector maximize the discounted present value of their cash flow, i.e., their revenue flow less their conversion and labor cost flows, by choosing the equilibrium input labor and deforestation rate for given cleared land dynamics.

Deforestation technology shows decreasing returns so that the deforestation cost is a convex function of the deforestation rate with a constant unit deforestation cost.

The aggregate supply of labor is fixed.

The economy can trade the two consumption goods and can borrow on a competitive capital market. The private and public sectors are aggregated so that the external constraint is automatically satisfied if each sector obeys its own intertemporal budget constraint (Burda and Wyplosz, 1993). Thus, since there are no assets in this economy, the change in the LDC foreign debt is given by the interests to be paid on the current debt minus the trade of balance surplus.

Individuals in the LDC obtain utility from consuming the two consumption goods according to two different preference indexes. Thus, the representative consumer maximizes the present value of his utility by choosing the equilibrium consumption levels of industrial and agricultural goods for given debt dynamics.

In this context, section 2 analyses the four differential equations system and derives the steady-state conditions for the stocks of forest and debt. As LDCs have shown an increasing stock of debt and a decreasing stock of forest, I have only considered initial conditions implying these dynamics.

Dynamic analysis for the stock of forest suggests that, provided its initial level is not extreme, if either the labor and cleared land productivities are sufficiently large or if the fraction of the national income spent on agricultural goods is sufficiently less than that spent on industrial goods, or if the forest regeneration rate and the unit deforestation costs are sufficiently large, then the stock of forest oscillates around an equilibrium level.

Dynamic analysis for the stock of debt indicates that it increases to
infinity when its initial level is sufficiently greater than its equilibrium level. However, the foreign debt will never be repudiated if the interest rate, and concern for future generations, are sufficiently small and large, respectively.

These results have provided a useful benchmark against which outcomes originating from the debt adjustment schemes are compared.

Section 3 discusses the DFNS’s impact on debt and forest dynamics.

In particular, within the framework developed above, subsection 3.1 provides the following insights. DFNSs make the stock of debt approach its equilibrium level, even if they do not revert the debt dynamics unless the fraction of the external debt converted through these schemes is sufficiently close to the interest rate. Moreover, DFNSs make the stock of forest approach its equilibrium level when its initial level is not extreme, whereas they may lead the forest to complete depletion when its initial level is very small; they do not revert the forest dynamics unless the reduction in the deforestation rate is sufficiently large. Finally, DFNSs are enforceable only if the LDC shows a small stock of forest.

In subsection 3.2 the validity of these insights is tested through some empirical evidence. It appears that the countries with extremely large stocks of cleared land have not signed DFNSs. The estimate for the reduction in the deforestation rate is consistent with the stocks of forest still decreasing in LDCs. Moreover, it turns out that the countries characterized by small stocks of cleared land have agreed DFNSs less than twice. The estimate for the fraction of the external debt converted through this scheme is consistent with the stocks of debt still increasing in LDCs. Finally, it transpires that the countries characterized by large stocks of cleared land have agreed DFNSs more than twice.

Section 4 analyzes the DFDS’s impact on debt and forest dynamics.

In particular, within the framework developed above, subsection 4.1 provides the following policy suggestions. The DFDS’s enforceability does not depend on the stock of forest so that DCs should be willing to agree these schemes with all LDCs. Moreover, DFDSs make the stock of debt approach its long-run equilibrium level to a smaller extent than DFNSs so that they are less likely to revert the debt dynamics. Thus only LDCs with small stocks of (or small growth rates in) debt should be willing to accept these schemes. Nevertheless, DFDSs reduce the stock of forest in the long-run equilibrium, apart from in countries characterized by an extremely small stock of forest, where they prevent its complete depletion. Thus only LDCs with very small stocks of forest should be involved in these schemes.

Therefore, the insights obtained in subsection 3.1, together with the policy suggestions achieved in subsection 4.1, indicate that a DFDS should be coupled with a DFNS in LDCs characterized by small stocks of forest and debt, and a DFDS may be adopted separately for LDCs characterized by small stocks of (or small growth rates in) debt and by stocks of forest that are not small.

In subsection 4.2 the application of these policy suggestions is assessed through some empirical evidence. It is shown that the countries characterized by large stocks of cleared land have agreed both a DFDS and a DFNS, those with small stocks of (and small growth rates in) debt and cleared

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land have agreed only a DFDS, and the countries characterized by extremely small stocks of cleared land have agreed only a DFDS. However, one country characterized by a large stock of cleared land has signed several DFDSs.

Section 5 concludes the paper.

1. The less developed country economy

Consider a LDC economy where two final goods are produced: agricultural good $X$ and industrial good $Y$. Producing the industrial good $Y$ requires labor $L_Y$ only. The production function of $Y$ is:

$$ Y = L_Y^\gamma $$

Thus firms in sector $Y$ maximize their profit by choosing the labor level:

$$ \max_{L_Y} L_Y^\gamma - WL_Y $$

where $P_Y$ is normalized to 1 and $W$ is the wage rate. Thus the following notional labor demand and profit levels prevail in equilibrium:

$$ L_Y = \left( \frac{\gamma}{W} \right)^{1/(1-\gamma)} $$

$$ \pi_Y = (1 - \gamma) \left( \frac{\gamma}{W} \right)^{\gamma/(1-\gamma)} $$

By contrast, producing the agricultural good $X$ requires labor $L_X$ and cleared land $C$. The production function of $X$ is:

$$ X = L_X^\alpha C^\delta $$

The land in the LDC is split into forest $F$, cleared land $C$, and other land. For the sake of simplicity let us disregard the latter and normalize to 1 the amount of land so that:

$$ F + C = 1 $$

Without loss of generality the standard logistic form for the regeneration function can be assumed to be:

$$ F - F^\delta \text{ with } \delta > 1 $$

Therefore, a decline in the forest occurs whenever the deforestation rate $H$ exceeds regeneration:

$$ \dot{F} = F - F^\delta - H $$

Alternatively, a decline in the cleared land $C$ occurs whenever regeneration exceeds the deforestation rate $H$:

$$ \dot{C} = -(1 - C) + (1 - C)^\delta + H $$

For the sake of simplicity let us assume that the deforestation technology shows decreasing returns so that, without loss of generality, the deforestation cost can be assumed to be:

$$ C_H = \frac{1}{2} \rho H^2 $$
with $\rho$ the constant unit deforestation cost.

This paper focuses on the relationship between the level of external debt and deforestation so that well-defined property rights on land are assumed to exist.

Therefore firms in sector $X$ maximize the discounted present value of their cash flow, i.e., their revenue flow less labor and deforestation cost flow, by choosing the labor and deforestation rate levels:

$$\max_{L_X, H} \int_0^\infty (P_X L^a X C^b - WL_X - \frac{1}{2} \rho H^2) e^{-\sigma t} dt$$

s.t. $\dot{C} = -(1 - C) + (1 - C)\delta + H$

where $P_X$ is the price of $X$ and $\sigma$ the time discount rate.

Thus the following dynamics for cleared land and its shadow price prevail in equilibrium (see appendix 1):

$$\dot{C} = -(1 - C) + (1 - C)\delta + \frac{m_c}{\rho}$$

$$m_c = -m_c[1 - \delta(1 - C)^{\delta-1}] - \beta P_X^{1/(1-\alpha)} \left( \frac{\alpha}{W} \right)^{\alpha/(1-\alpha)} C^{(\alpha-\beta-1)/(1-\alpha)}$$

$$\lim_{t \to \infty} m_c e^{-\sigma t} \geq 0$$

$$\lim_{t \to \infty} m_c e^{-\sigma t} = 0$$

Notice that the assumption of well-defined property rights implies that firms in sector $X$ take into account the future deforestation costs so that the shadow price of cleared land is positive.

Let the labor available to the economy be limited by:

$$L_X + L_Y = 1$$

so that the wage rate $W$ represents also the national labor income. The equilibrium condition ensures:

$$I = P_X X + Y$$

where $I$ represents the national income. Thus the budget constraint for the economy is given by:

$$\pi_X + \pi_Y + W = P_X X + Y$$

where $\pi_X$ and $\pi_Y$ represent the profits in sector $X$ and $Y$ respectively.

The final goods may either be consumed directly or traded. Let $E_X$ refers to exports of $X$ and $M_Y$ to imports of $Y$. LDCs imports can be funded through borrowing from DCs commercial banks at a competitive interest rate $i$.

This paper concentrates on the LDC external debt so that the private and public sectors are aggregated. Both DFNSs and DFDSs can be seen as mechanisms that enable countries facing credit constraints to return to an unrestricted credit regime. This paper deals with countries having signed these swaps so that no binding credit constraints are assumed to prevail.

Since there is no capital in this economy, the balance of payment identity is given by:

$$\dot{D} = iD + M_Y - r P_X E_X$$
where $\dot{D}(t)$ is the net flow of debt in period $t$ as a result of new borrowing less repayment of principal, $iD(t)$ is the interest payments in period $t$, and $r$ is the exchange rate.

Notice that the aggregation of private and public sectors implies that the external constraint is automatically satisfied if each sector obeys its own intertemporal budget constraint. Moreover, the assumption of the absence of a binding credit constraint implies that the separation theorem applies.

Since there is no capital in this economy, only borrowing for consumption is considered. The extension of the analysis to incorporate a productive role for capital would have little effect. Without any adjustment costs associated with investments the stock of capital would be brought to the level at which its marginal product equals the world interest rate simultaneously: it would be sufficient to redefine the foreign debt level in order to take into account the investments, and the analysis would follow as before.

When LDCs import $X$ and export $Y$, $E_X$ and $M_Y$ are both defined to be negative. Thus, by definition, the consumption level of $X$ is given by $X - E_X$ and the consumption level of $Y$ is given by $Y + M_Y$.

LDCs are populated by identical individuals. The representative consumer obtains utility from consuming the two final goods, the agricultural good $X$ and the industrial good $Y$. Therefore LDC agents maximize the discounted present value of their utility from consumption by choosing the demand and the export levels for the agricultural goods and the import levels for the industrial goods:

$$\max_{X, E_X, M_Y} \int_0^\infty (X - E_X)^A(I - P_XX + M_Y)^B e^{-at} dt$$

s.t. $\dot{D} = iD + M_Y - RE_X$

where the export price $R = rP_X$ is assumed to be equal to the domestic price $P_X$ for the arbitrage condition.

Notice that the Cobb–Douglass utility function implies that $A/(A + B)$ and $B/(A + B)$ represent the fraction of national income spent on agricultural and industrial goods, respectively. Consequentially, $A > B$ is equivalent to the former being greater than the latter.

Thus the following dynamics for debt and its shadow price prevail in equilibrium (see appendix 2):

$$\dot{D} = iD - I + (1 + \frac{A}{B}) \left( \frac{B}{m_D} \right)^{1/(1-A-B)} m_D$$

$$\lim_{t \to \infty} m_D e^{-at} \geq 0$$

$$\lim_{t \to \infty} m_D D e^{-at} = 0$$

Notice that the aggregation of private and public sectors implies that
agents in LDC take into account the future debt burden costs so that the shadow price of debt is positive.

2. The debt and cleared land dynamics
The purpose of this paper is to analyze the impact on the stock of forest and the stock of debt in LDCs of some debt adjustment schemes. In this section the two differential equations for stock of debt and stock of cleared land will be derived from the model introduced above and the results obtained from the analysis of their dynamics will provide a useful benchmark against which outcomes originating from the debt adjustment schemes will be compared.

Three markets exist in the LDC economy. Walras’ law allows us to focus on the equilibrium conditions for two of them only.

Consumers demand any quantity of X provided the domestic price ratio equals the international price ratio (see appendix 2): \( P_X^* = R \). In order to simplify the notation, let the international price ratio equal the preferences index ratio: \( R = A/B \). Consequently, the quantity of agricultural good exchanged in equilibrium is defined by the quantity supplied at this equilibrium price (see appendix 1):

\[
X^* = \left( \frac{A}{B} \right)^{\alpha/(1-\alpha)} \left( \frac{\alpha}{W} \right)^{\alpha/(1-\alpha)} C^{\beta/(1-\alpha)}
\]

Moreover, the equilibrium in the labor market is ensured by the wage rate \( W \) equalizing demand for and supply of labor:

\[
\left( \frac{A}{B} \right)^{1/(1-\alpha)} \left( \frac{\alpha}{W} \right)^{1/(1-\alpha)} C^{\beta/(1-\alpha)} + \left( \frac{\gamma}{W} \right)^{1/(1-\gamma)} = 1
\]

Alternatively, the equilibrium in the industrial goods market is ensured by the wage rate \( W \) satisfying \( W = \gamma Y + \alpha P_X^* X^* \) and \( Y = (\gamma/W)^{\gamma/(1-\gamma)} \). It is easy to check that these conditions coincide with the condition introduced above.

Unfortunately this condition cannot be solved explicitly with respect to \( W \) unless \( \alpha = \gamma \). In order to simplify calculations let this circumstance occur. Thus, the wage rate prevailing in equilibrium is given by:

\[
W^* = \alpha \left[ 1 + \left( \frac{A}{B} \right)^{1/(1-\alpha)} C^{\beta/(1-\alpha)} \right]^{(1-\alpha)}
\]

Therefore the following four equations form a dynamic system together with the following four transversality conditions describe the LDC economy:

\[
\dot{C} = -(1 - C) + (1 - C)^{\delta} + \frac{m_c}{\rho}
\]
\[
\dot{m}_C = -m_c[1 - \delta(1 - C)^{b-1}] - \beta \left( \frac{A}{B} \right)^{1/(1-a)} \left[ 1 + \left( \frac{A}{B} \right)^{1/(1-a)} C^{\beta/(1-a)} \right]^{(-\alpha)} C^{(\alpha+\beta-1)/(1+\alpha)}
\]

\[
\lim_{t \to \infty} m_c e^{-\alpha t} \geq 0
\]

\[
\lim_{t \to \infty} m_c e^{-\alpha t} = 0
\]

\[
D = iD - \left[ 1 + \left( \frac{A}{B} \right)^{1/(1-a)} C^{\beta/(1-a)} \right]^{(1-a)} + \left( 1 + \frac{A}{B} \right) \left( \frac{B}{m_D} \right)^{1/(1-\alpha-\beta)}
\]

\[
\dot{m}_D = i m_D
\]

\[
\lim_{t \to \infty} m_D e^{-\alpha t} \geq 0
\]

\[
\lim_{t \to \infty} m_D D e^{-\alpha t} = 0
\]

From equations (1) and (2) the cleared land dynamics is obtained. The invariant curve \( \dot{\mathcal{C}} = 0 \) and \( \dot{m}_C = 0 \) are given by:

\[
m_C = \rho[(1 - C) - (1 - C)^{b}]
\]

\[
m_c = -\frac{1}{1 - \delta(1 - C)^{b-1}} \beta \left( \frac{A}{B} \right)^{1/(1-a)} \left[ 1 + \left( \frac{A}{B} \right)^{1/(1-a)} C^{\beta/(1-a)} \right]^{(-\alpha)} C^{(\alpha+\beta-1)/(1+\alpha)}
\]

Let us call \( I_c \) and \( I_{mc} \) the right-hand sides of the first and second of these expressions respectively.

Notice that \( I_c \) first increases and then decreases with an absolute maximum for \( C = 1 - \delta^{1/(1-\beta)} \). Moreover, \( I_{mc} \) shows two vertical asymptotes for \( C = 0 \) and \( C = 1 - \delta^{1/(1-\beta)} \) if \( \alpha + \beta < 1 \) and a single asymptote for \( C = 1 - \delta^{1/(1-\beta)} \) otherwise.

The dynamic analysis allows us to state:

**Proposition 1** Equations (1) and (2) show a bifurcation such that no equilibrium, one unstable equilibrium and two equilibria (a saddle and a centre) exist for \( r, r^*, r_5, r > r^* \), respectively, with \( r^* = -I_{mc}/[(1 - C) - (1 - C)^{b}] \).

Proof. See appendix 3.

A numerical experiment with \( \alpha = 0.7, \beta = 0.3, A = 0.28, B = 0.72, \delta = 2, \rho = 2.11, m_c(0) = 0.8, C(0) = 0.2 \) is provided in figure 1.

Since LDCs have shown a decreasing stock of forest, only \( m_c(0) \) and \( C(0) \) such that \( \dot{\mathcal{C}} > 0 \) will be considered.

Therefore, if either the labor and cleared land productivities are sufficiently small (small \( \alpha \) and small \( \beta \)) or if the fraction of the national income spent on agricultural goods is sufficiently greater than that spent on industrial goods (\( A > B \)) or if the forest regeneration rate \( \delta \) and the unit deforestation cost \( \rho \) are sufficiently small, then the stock of forest is destined to disappear completely.
Indeed, if the low productivity of inputs implies a production level in the agricultural sector inadequate for the consumption pattern, then, provided the deforestation cost is small enough, the exploitation of the forest will lead to its complete depletion unless the reforestation rate is sufficiently great. However, the concomitance of these features seems unlikely to occur.

Otherwise, three regions are identified.

Indeed, if the deforestation cost is sufficiently large, the relative national income spent on agricultural goods is sufficiently small, and the cleared land and labor productivities are sufficiently high, then the deforestation rate can be larger or smaller than the reforestation rate according to the initial stock of forest. In particular, the forest will vanish completely when its initial level is very small, and it will oscillate around an equilibrium level when its initial level is not extreme.

From equations (3) and (4) the debt dynamics is obtained. The invariant curve $D = 0$ is given by:

$$D = \frac{1}{i} \left\{ \left[ 1 + \left( \frac{A}{B} \right)^{1/(1-\alpha)} C^0/(1-\alpha) \right]^{(1-\alpha)} - \left( 1 + \frac{A}{B} \left( \frac{B \exp[-it]}{m_D(0)} \right)^{1/(1-A-B)} \right) \right\}$$

Let us call $I_d$ the right-hand side of this expression. The dynamic analysis allows us to state:

**Proposition 2** Provided the initial stock of debt $D(0)$ is sufficiently large, equations (3) and (4) show a stock of debt infinitely increasing above an invariant curve with the following properties: it increases in cleared land; it decreases in time if $A + B < 1$ and $\partial C/\partial t = C' > 0$; it is concave in cleared land if and only if $\alpha + \beta < 1$; it is concave in time if $\partial^2 C/\partial t^2 = C'' < 0$.

Proof. See appendix 4.

Therefore, the stock of debt in equilibrium will be larger, the larger the
stock of cleared land \( \partial I_i/\partial C > 0 \) is, even if this positive relationship decreases \( \partial^2 I_i/\partial C^2 < 0 \) when decreasing returns to scale prevail in the agricultural sector \( (\alpha + \beta < 1) \). Indeed, an increase in the cleared land implies an increase in agricultural production and exports so that a larger stock of debt in equilibrium can be afforded. However, in the presence of decreasing returns to scale, a given increase in the cleared land will cause progressively lower increases in agricultural production and exports so that the positive relationship between the stock of debt in equilibrium and the stock of cleared land decreases.

Moreover, the stock of debt in equilibrium will become smaller over time \( \partial I_i/\partial t < 0 \), and this negative relationship increases \( \partial^2 I_i/\partial t^2 > 0 \) when a proportional increase in the consumption of both agricultural and industrial goods leads to a less than proportional increase in the utility level of the representative consumers \( (A + B < 1) \). Thus the stock of cleared land increases in time \( (C' > 0) \) at a decreasing rate \( (C'' < 0) \). Indeed, as time passes, the stock of debt in equilibrium decreases because of interest payments. If the stock of cleared land increases, agricultural production and exports also increase, even if the latter increases to a lesser extent because of the preference structure. Moreover, if the increase in the stock of cleared land decreases over time, the export increase also decreases over time, so that the negative relationship between the stock of debt in equilibrium and time increases.

Since LDCs have shown an increasing stock of debt, only \( m_D(0) \) and \( D(0) \) such that \( \dot{D} > 0 \) will be considered. This implies that the stock of debt will increase infinitely.

This is what has almost always happened since the 1970s in LDCs.

One must ask, however, whether this dynamic can persist over time.

Almost all sovereign borrowers are solvent in the sense that the discounted present value of their national resources exceeds the value of their external debt (Eaton, 1993). The solvency restriction can be assumed to hold. Moreover, debt has not witnessed a growth rate, on average, faster than the international interest rate (Eaton, 1993). The trasversality condition seems to be met.

Commercial banks, however, will restrict the LDC to debt and repayment profiles that satisfy the enforcement constraint, i.e., it is never in the interest of the LDC to repudiate its foreign debt.

Eaton and Gersovitz (1981), Grossman and van Huyck (1988), or Kletzer (1984) have shown that whenever default implies not only that foreign lenders will not lend to the borrower again but that foreign investments by the borrower will not be repaid, lending and repayment can be sustained. I will assume that default leads to total financial autarch, not just to an embargo on gross loans.

Unlike the above models, a deterministic framework has been developed here. I will assume that \( D(t) \) is observable and refers to an average stock of debt over the period \( (t, t-1) \).

Individuals in the LDC are better off by repudiating the foreign debt at time \( t_r \) if the discounted value of current and future utility with access to international financial market (to borrow and lend) is smaller than under financial autarchy.
A welfare analysis allows us to state:

**Proposition 3** Provided \( [i + \sigma(1 - A - B)]/(1 - A - B) > 0 \), the foreign debt will not be repudiated at time \( t_r \), if the following condition holds:

\[
t_r < -\frac{\sigma}{i + \sigma(1 - A - B)} \log \left[ Z^{-\omega}[Z - z] \left( \frac{m_{D,0}}{B} \right)^{1/(1 - A - B)} \right] \times \frac{i + \sigma(1 - A - B)}{\sigma(1 - A - B)}
\]

where: \( Z = 1 + (A / B)^{1/(1 - \omega)} C^\beta / (1 - \omega) \) and \( z = [(A / B)C^\beta]^{1/(1 - \omega)} \).

Proof. See appendix 5.

Therefore, if the concern for future generations is sufficiently large (small \( \sigma \)) and the interest rate in the capital market is sufficiently small (small \( i \)), the external debt will never be repudiated. Indeed, the external debt allows a country to increase the consumption level through imports, on the one hand, and forces it to divert a fraction of national income into interest payments, on the other hand. If at each time the value attached to the internal resources devoted to interest payments is smaller than the value attributed to the greater consumption that the external debt would allow future generations to achieve, repudiation will never occur.

Solutions obtained above stress the importance of initial conditions. Therefore the knowledge of current stocks of cleared land and debt together with the technology and preference structure is essential to identify the LDC economy in the \((D, C)\) space and to analyze the dynamics being considered in this paper.

In next sections the impacts of DFNSs and DFDSs on the dynamics highlighted in this section will be analyzed.

### 3. The debt-for-nature swaps

The bargaining problems underlying the LDCs debt crisis have been under intense investigation. On the one hand, new institutions have been recommended. In particular, the creation of a new multilateral lending institution to buy up discounted debt and pass the discounts on to the debtor country (Kenen, 1990; Sachs, 1990) and multilateral aid directed at rewarding good behavior by the debtor country (Bulow and Rogoff, 1990) have been suggested. The effectiveness of these suggestions has been challenged on several grounds (Eaton, 1990). On the other hand, various schemes have been proposed in which the debtor countries buy back their debt on the secondary market, with buybacks taking the form of market buybacks and swap arrangements. In particular, the extent to which market buybacks benefit debtor countries (Bulow and Rogoff, 1988) and the problem of free-riding in the presence of heterogeneity across banks (Diwan and Kletzer, 1992) have been discussed. Several swap arrangements schemes have been devised (Krugman, 1989).

The bilateral agreement surrounded by the greater enthusiasm and optimism for solving both the environmental and debt problems in LDCs is the DFNS. This is a kind of debt–equity swap involving the purchase at dis-
counted value in the secondary market of a LDC debt and its cancellation in return for investments in environmental projects such as management and protection of nature reserves.

The impact of this scheme on the forest and debt dynamics in the context of the model developed above and some empirical evidence will be presented in the next subsections.

3.1 The impact

A DFNS consists of an external and an internal process.

The external process involves an international bank or institution willing to negotiate part or all of its loan to a specific LDC at below face value, and an international donor or supportive government interested in conservation projects in the debtor country and willing to buy part of the debt in the secondary market at a substantial discount to its benefit and to that of the beneficiary country and organizations.

Let us call $0 < \lambda_{fn} < 1$ the fraction of the external debt converted through the DFNS.

The internal process involves the central government (minister of the environment or of the natural resources), the central bank, or financial authority and the national non-government organization. The negotiations between them determines the maximum eligible amount of debt at nominal value; the percentage recognition of the external debt for conversion into local currency government bonds for the national beneficiary organization; the bond maturity; the interest rate; the procedure to be followed; and the organization involved in investments in projects such as wildland conservation and forestry schemes, land purchase, technical support, etc.

Two main purposes have been indicated in the general statements of the DFNSs agreed upon so far.

Sometimes they aim at increasing parks and nature reserves. In this case the DFNS could be depicted by introducing a parameter representing the fraction of cleared land converted into forest through it. Sometimes they aim at enhancing the delineation and enforcement of existing nominal property rights held by the LDC government. In this case the DFNS could be depicted by introducing a parameter representing the reduction in the deforestation rate obtained through it.

A quick look at the specific projects funded by DFNSs shows that the second purpose is more often pursued. Therefore the second representation is adopted.

Let us call $0 < \mu_{fn} < 1$ the reduction in the current deforestation rate implied by the DFNS.

Notice that if the DFNS takes the form of compensation that can be stopped if the forest management agreement is not strictly adhered to (Hansen, 1989), it can be depicted by assuming a positive relationship between $\lambda_{fn}$ and $\mu_{fn}$.

If the debt reduction is assumed to be spent in the same period as that in which it occurs, the DFNS can be represented by a couple $\lambda_{fn}$ and $\mu_{fn}$ such that:

$$\dot{D} < 0 \Leftrightarrow D < \frac{i}{i - \lambda_{fn}} I_d$$

(5)
\[
\dot{C} < 0 \iff m_c < \frac{1}{1 - \mu_{fn}} I_c \tag{6}
\]
where \(0 < \lambda_{fn} < 1\) and \(0 < \mu_{fn} < 1\).

A straightforward analysis of the dynamic system allows us to state:

**Proposition 4** For each \(m_{c(0)} > 0\), \(D(0) > I_d\), and \(0 < C(0) < 1\), there exists \(\lambda_{fn}\) with \(1 > \lambda_{fn} > 0\) such that \(D(0) < 0\). By contrast, for some \(m_{c(0)} > I_c(0)\) and \(0 < C(0) < 1\), there is no \(\mu_{fn}\) with \(\mu_{fn} > 1\) such that \(C(0) < 0\) and \(1 > \lim_{t \to \infty} C > 0\).

Proof. It is easy to check that if \(\lambda_{fn}\) tends to \(i\) then \(i/(i - \lambda_{fn})\) tends to infinity and (5) holds. By contrast, (i) if \(0 < C(0) < 1 - \delta^{1/(1-\delta)}\), then the following conditions must hold: \(I_c(0)/(1 - \mu_{fn}) > m_c(0)\) and \(I_{mc}(0) < m_c(0)\); (ii) if \(1 - \delta^{1/(1-\delta)} < C(0) < 1\), then the following conditions must hold: \(I_c(0)/(1 - \mu_{fn}) > m_c(0)\) and \(I_c(0) < m_c(0)\). It easy to check that if \(\mu_{fn}\) tends to \(1\), then \(1/(1 - \mu_{fn})\) tends to infinity and (6) holds. However, in case (i) the condition \(I_{mc}(0) < m_c(0)\) may not be met if \(C(0)\) is sufficiently close to \(0\); in case (ii) the condition \(I_{c(0)} < m_c(0)\) may not be met for \(C(0)\) sufficiently close to \(1\).

Therefore, if the reduction in the stock of debt \(D\) is sufficiently close to the interest rate \((\lambda_{fn} \text{ close to } i)\) so that the payments of the external debt \(iD\) become sufficiently small, then the debt dynamics is reversed. If the stock of cleared land is extremely small, the reduction in the deforestation rate required for its dynamics to be reversed may be so large that the relative dynamics of the stock of forest becomes larger than the relative dynamics of the deforestation rate \((\partial \hat{F}/\partial \hat{H} < \partial \hat{H}/\hat{H})\) and, consequently, the cleared land disappears eventually. By contrast, if the stock of cleared land is small, the reduction in the deforestation rate may reverse its dynamics, but this effect does not persist over time. Analogously, if the stock of cleared land is large, the reduction in the deforestation rate required for its dynamics to be reversed may be so large that the relative dynamics of the stock of forest becomes smaller than the relative dynamics of the deforestation rate \((\partial \hat{F}/\partial \hat{H} < \partial \hat{H}/\hat{H})\) and, consequently, the cleared land eventually covers the entire land area.

Several insights can be drawn from these results. Apart from countries characterized by extreme stocks of forest, the DFNSs show potentials.

Indeed, they make the forest and debt stocks approach their equilibrium levels. In the context developed above, this is equivalent to a reduction in the stock of debt \(D\) towards the debt invariant curve \(I_d\) and a reduction in the cleared land shadow price \(m_c\) and, consequentially, a contraction of the oscillations around the centre. This result seems to suggest the absence of LDCs characterized by extreme stocks of cleared land from those with whom DCs are willing to agree these schemes.

However, the DFNSs reveal limitations as well.
First, they do not revert the forest stock dynamics unless the reduction in the deforestation rate is sufficiently large. In the context developed above, this implies reducing the cleared land shadow price $m_c$ below the cleared land invariant curve $I_c$. This result predicts forest stocks still decreasing in LDCs.

Second, the DFNSs do not revert the debt-stock dynamics unless the fraction of the external debt converted through these schemes is close to the interest rate. Within the framework introduced above, this implies reducing the stock of debt $D$ below the debt invariant curve $I_d$. This result predicts debt stocks still increasing in LDCs.

Third, the DFNSs may lead to the complete depletion of the stock of forest when its initial level is quite small. In the context developed above, this is depicted by a cleared land shadow price $m_c$ below the null eigenvalues curve $I_e$ and above the cleared land invariant curve $I_c$. This result seems to suggest the absence of LDCs characterized by a quite large stock of cleared land from those DCs willing to stipulate for these schemes.

Fourth, the DFNSs may lead the stock of forest to its utmost expansion when its initial level is quite large. Within the framework introduced above, this is depicted by a cleared land shadow price $m_c$ below the null eigenvalues curve $I_e$ and above the cleared land invariant curve $I_c$. This result leads us to predict the absence of LDCs characterized by a quite small stock of cleared land from those willing to accept these schemes.

Fifth, the DFNSs are enforceable only when the stock of forest is small. In the context developed above, this is depicted by a cleared land shadow price $m_c$ above the null eigenvalues curve $I_e$ and the cleared land invariant curve $I_c$. This result predicts the prevalence of LDCs characterized by large stocks of cleared land with whom DCs are repeatedly willing to agree these schemes.

Notice that the willingness to agree upon the DFNS is deduced only from the consistency of the LDC optimal cleared land and debt dynamics with the dynamics implied by the swap. Indeed, a welfare analysis would not be carried out because of the complexity of these stocks dynamics.

Therefore, the insights obtained in this subsection can be summarized as follows. The LDCs characterized by large, small, or extreme stocks of forest are likely to agree the DFNS a few times, several times, or never, respectively.

These issues are discussed empirically in the next subsection.

3.2 Some empirical evidence

The analysis developed above gave us several interesting insights. Their validity will be tested in this subsection.

Unfortunately, all data on environmental resources must be treated with caution and data used in this section are no exception. Although they are indicative of major differences in resource endowments and uses between countries, true compatibility is limited because of variations in data collection, statistical methods, definitions, and government resources.

Since an international donor willing to buy part of the debt on the secondary market and interested in conservation projects in the debtor country must be present, countries with low annual deforestation rates or
Large forest stocks will not be considered. Let us take the following threshold values: 1 per cent average annual deforestation rates over the 1980s and 30 per cent of forest and woodland in 1986. These criteria allow me to focus on 19 countries only. The first column of table 1 presents total external debt over GNP in 1986, where total external debt is defined as the long-term debt, use of the IMF credit, and the short-term debt (World Bank, 1997a); its average annual growth rate between 1980 and 1986 is recorded in the second column (World Bank, 1997a). The third column of table 1 presents the cleared land in 1986 defined as arable land, land under permanent crops, and permanent meadows and pastures (FAO, 1996); its average annual growth rate during 1980s is recorded in the fourth column (World Bank, 1997b). The fifth and the sixth columns provide the number of DFNSs and DFDSs agreed, respectively.

Therefore, the insights obtained by the analysis developed in the previous subsection seem to be supported by empirical evidence.

First, countries with extremely large stocks of cleared land that have not signed the DFNS in spite of their large deforestation rates are Bangladesh, Haiti, Pakistan, and El Salvador.

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>$D$</th>
<th>$\dot{D}$</th>
<th>$C$</th>
<th>$\dot{C}$</th>
<th>DFN</th>
<th>DFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Madagascar</td>
<td>107.6</td>
<td>2.46</td>
<td>0.71</td>
<td>0.9</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>51.8</td>
<td>0.59</td>
<td>0.82</td>
<td>4.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Haiti</td>
<td>31.9</td>
<td>0.53</td>
<td>0.97</td>
<td>6.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Nigeria</td>
<td>60.5</td>
<td>4.99</td>
<td>0.86</td>
<td>0.8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Zambia</td>
<td>417.3</td>
<td>3.60</td>
<td>0.55</td>
<td>1.1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Ghana</td>
<td>49.1</td>
<td>0.55</td>
<td>0.48</td>
<td>1.4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Pakistan</td>
<td>43.1</td>
<td>0.02</td>
<td>0.89</td>
<td>4.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bolivia</td>
<td>156.0</td>
<td>0.67</td>
<td>0.34</td>
<td>1.3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Philippines</td>
<td>96.4</td>
<td>0.79</td>
<td>0.52</td>
<td>4.0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Guatemala</td>
<td>39.8</td>
<td>1.67</td>
<td>0.47</td>
<td>1.9</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>71.1</td>
<td>1.28</td>
<td>0.85</td>
<td>3.3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Ecuador</td>
<td>90.1</td>
<td>0.67</td>
<td>0.38</td>
<td>2.0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>El Salvador</td>
<td>46.0</td>
<td>0.76</td>
<td>0.93</td>
<td>2.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Jamaica</td>
<td>192.6</td>
<td>1.47</td>
<td>0.72</td>
<td>11.2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>110.7</td>
<td>0.85</td>
<td>0.63</td>
<td>3.5</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Poland</td>
<td>51.4</td>
<td>n.a.</td>
<td>0.68</td>
<td>-0.1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Panama</td>
<td>94.4</td>
<td>0.15</td>
<td>0.40</td>
<td>2.1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Brazil</td>
<td>44.4</td>
<td>0.42</td>
<td>0.34</td>
<td>0.7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Mexico</td>
<td>82.9</td>
<td>1.72</td>
<td>0.70</td>
<td>1.4</td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: $D$ is the total external debt over GNP in 1986, with total external debt defined as the long-term debt, use of the IMF credit, and the short-term debt; $\dot{D}$ is its average annual growth rate between 1980 and 1986 (World Bank, 1997a). $C$ is the cleared land in 1986 defined as arable land, land under permanent crops, and permanent meadows and pastures (FAO, 1996); $\dot{C}$ is its average annual growth rate during 1980s (World Bank, 1997b). DFN and DFD are the number of DFNSs and DFDSs agreed, respectively.
The estimate for $\mu_{fn}$ is 0.002 and, consistent with the model developed above, the stocks of forest are still decreasing in LDCs.

Second, countries characterized by large stocks of cleared land that have agreed the DFNS more than twice are Madagascar, Philippines, Costa Rica, and Mexico. A further country should belong to this group. However, its absence can be explained by noting that Ecuador, in spite of its small stock of cleared land, shows 39.2 per cent of protected area indicating that swaps have been devoted to delineate and enforce the existing property rights held by the government.

The estimate for $\lambda_{fn}$ is 0.001 and, consistently with the model developed above, the stocks of debt are still increasing in LDCs.

Third, countries with small stocks of cleared land that have agreed the DFNS less than twice are Ghana, Bolivia, Guatemala, Panama, and Brazil. More countries should belong to this group. However, the absence of Nigeria and Poland can be explained by noting that these countries show small deforestation rates that DCs were unlikely to be interested in reducing. On the other hand, the absence of Zambia, the Dominican Republic, and Jamaica can be accounted for by observing that these countries have experienced high deforestation rates with respect to their stocks of forest and, consequently, DCs were more likely to preferentially sign alternative swaps with these countries.

These results are consistent with the estimates of coefficients for the tropical land share and for the threatened species index obtained by Deacon and Murphy (1997) in a probit model and a tobit model assessing, respectively, the probability of a DFNS occurring and of the number of DFNSs completed.

Therefore, the following conclusions can be drawn from this section.

LDCs have recognized the expedience of signing the DFNS, their motivation being the debt burden issue. Indeed, these swaps have actually reduced the stocks of debt. DCs have seized the opportunity to stipulate the DFNS with any LDC characterized by a stock of forest which is not extreme, their motivation being the tropical deforestation issue. Indeed, these swaps have actually reduced the deforestation rates. In other words, the conditions for effectiveness always hold.

However, it appears that the conditions for enforceability have been met only in countries characterized by a small stock of forest, where both the consistency of the DFNS’s projects and the LDCs self-interest prevail.

In the next section an alternative scheme is considered.

4. The debt-for-development swaps

In recent years, several kinds of debt reduction schemes involving the cancellation of the LDC debt have been suggested.

The debt-for-cash swap allows the debtor country to repurchase its debt at a discount; the debt-for-equity swap converts external debt into local currency equity in a domestic firm; the debt-for-bonds swap allows banks to convert their loans into low interest rate bonds exempted from future concerted lending and new money calls. The financial nature of these schemes makes it difficult to assess their impact on the forest stock dynamics.

Moreover, the debt-for-export swap implies the cancellation of the
LDCs’ debt in return for investments in non-traditional exports such as textiles, metal alloys, gold and silver jewelry, frozen fish, processed food and chemicals. The limited list of products exported through this scheme leads to the conclusion that their impact on forest dynamics is negligible.

Finally, the DFDS implies the cancellation of the LDCs’ debt in return for investments in development projects such as health and education projects.

The impact of this scheme on the forest and debt dynamics in the context of the model developed above and some empirical evidence will be presented in the following subsections.

4.1 The impact

As shown above for the DFNS, the fraction of external debt converted through the DFDS is called \( \lambda_{fd} \).

Analogously, the increase in labor productivity implied by this scheme is depicted by considering a change in the parameter \( \alpha \). Let us call \( \alpha_{fd} \) the productivity of labor implied by the DFDS.

Therefore, if the debt reduction is assumed to be spent in the same period as that in which it occurs, the DFDS can be represented by a couple \( \lambda_{fd} \) and \( \alpha_{fd} \) such that:

\[
\begin{align*}
\dot{D} < 0 & \iff D < \frac{i}{i - \lambda_{fd}} I_d(\alpha_{fd}) \\
\dot{C} > 0 & \iff m_c > I_c \\
\dot{m}_c > 0 & \iff m_c > I_{mc}(\alpha_{fd})
\end{align*}
\]

where \( 0 < \lambda_{fd} < 1 \).

A straightforward analysis of the dynamic system allows us to state:

**Proposition 5** For each \( m_c(0) > 0 \) and \( D(0) > I_d \), there exists a \( \lambda_{fd} \) with \( 1 > \lambda_{fd} > 0 \) such that \( D(0) > 0 \). Moreover, the invariant curves \( I_{mc} \) and \( I_d \) are decreasing in \( \alpha_{fd} \).

Proof. It is easy to check that if \( \lambda_{fd} \) tends to \( i \) then \( i/(i - \lambda_{fd}) \) tends to infinity and (7) holds. Moreover:

\[
\begin{align*}
\frac{\partial I_{mc}}{\partial \alpha_{fd}} < 0 & \iff - \log \left[ \frac{z^{1/(1-\alpha)}}{1 + z} \right] > z \log \left[ \frac{z}{1 + z} \right] \\
\frac{\partial I_d}{\partial \alpha_{fd}} < 0 & \iff z \log \left[ \frac{z}{1 + z} \right] < \log [1 + z]
\end{align*}
\]

where \( z = [(A/B)^{\beta}]^{1/(1-\alpha)} \). These conditions are always met.

Therefore, if the reduction in the stock of debt \( D \) is sufficiently close to the interest rate \( \lambda_{fd} \) close to \( i \) so that the payments on account of the external debt \( iD \) become sufficiently less, then debt dynamics are reversed. Moreover, an increase in the labor productivity (larger \( \alpha_{fd} \)) has two consequences in the long-run. On the one hand, it implies an increase in the national income for each amount of cleared land: this leads to an increase
in imports, a decrease in the primary account given the stock of debt, and, consequently, to a reduction in its equilibrium level (lower $I_d$). On the other hand, it implies an increase in the optimal cleared land to be applied in equilibrium (lower $I_{mc}$).

Several policy suggestions can be drawn from these results. The DFDSs show higher potentials than the DFNSs. Indeed, unlike the DFNSs, their enforceability does not depend on the stock of forest. In the context developed above, this is depicted by the absence of required dynamics for the cleared land. This result suggests that DCs should be willing to agree these schemes with all LDCs.

However, the DFDSs show limitations as well. Indeed, they make the stock of debt approach its long-run equilibrium to a lesser extent than the DFNSs. In the context developed above, this is depicted by a lower debt invariant curve $I_d$ for each stock of debt $D$. This result suggests that only LDCs with small stocks of (or small growth rates in) external debt should be willing to accept these schemes. Nevertheless, they reduce the stock of forest in the long-run equilibrium, apart from in countries that have an extremely small stock of forest, where they prevent its complete depletion. Within the framework introduced above, this is depicted by lower invariant curves $I_{mc}$ and $I_e$. This result suggests that only LDCs with an extremely large stock of cleared land should be involved in these schemes.

Notice that again the willingness to agree upon the DFDS is deduced only from the consistency of the optimal cleared land and debt dynamics with the dynamics implied by the swap. Indeed, a welfare analysis would not be carried out because of the complexity of these stocks dynamics.

Therefore, the policy suggestions obtained in this subsection can be summarized as follows. LDCs characterized by stocks of forest that are not extremely small, and with large stocks of (or large growth rates in) external debt, should not be involved in the DFDS.

The insights discussed in the previous section coupled with the policy suggestions introduced in this section lead to the conclusion that a DFDS can be adopted when the LDC is characterized by a small stock of forest provided a DFNS is also agreed. Indeed, the latter attenuates the negative impact on forest of the former. Moreover, a DFDS may be adopted separately when the LDC is characterized by a large or an extremely large stock of forest. Indeed, in this case its negative consequences on forest do not matter.

These issues are discussed empirically in the next subsection.

4.2 Some empirical evidence

The analysis developed above revealed several interesting policy implications. Their application will be assessed in this subsection.

Table 2 provides the data for countries that have signed the DFDS only. Therefore, in some cases the policy suggestions obtained by the analysis developed in the previous subsection seem to have been noted.

First, the countries characterized by large stocks of cleared land that have agreed both the DFDS and the DFNS are Madagascar, Nigeria, Zambia, Ghana, Bolivia, Philippines, Jamaica, and Mexico. Thus, the complementary nature of these swaps has been observed.
Second, the countries with small stocks of (and small growth rates in) debt that have agreed the DFDS only are: Senegal and Peru. Thus, the constraint on the stock of debt has been taken into account.

Third, the countries characterized by extremely small stocks of cleared land that have agreed the DFDS only are Tanzania, Kenya, and South Africa. Thus, the constraint on the stock of cleared land has been observed. However, in one case the policy suggestions obtained by the analysis developed in the previous subsection seem not have been followed. Indeed, in spite of being characterized by a large stock of cleared land, Sudan has signed several DFDSs, implying a relative increase in the productivity of labor because of the water, sanitation, and health education programs funded through it.

This result is consistent with the deforestation rate observed in Sudan over the last two decades, largely as a result of the expansion of rainfed mechanized farming (Bromley, 1992).

Therefore, the following conclusions can be drawn from this section. The conditions for enforceability always hold because they do not depend on the stock of forest.

However, because of the debt burden issue, only LDCs with a small stock of debt should accept a DFDS. Moreover, because of the tropical deforestation issue, DCs should sign a DFDS when LDCs are characterized by an extremely small stock of forest. In other words, the conditions for effectiveness turn out to be met only when the LDC is characterized by an extremely small stock of forest and a small stock of debt.

Nevertheless, a LDC with a small stock of forest can be involved in a DFDS provided a DFNS is also agreed, and a LDC with a large or an extremely large stock of forest can participate to a DFDS because of the lack of worry about depletion of its forest.

5. Concluding remarks

The dynamics of debt and forest in a LDC have been examined in an international framework using a four differential equations system.

The debt dynamics are obtained by maximizing the present value of utility from consumption of industrial and agricultural goods for given interest to be paid on the current debt and balance of trade surplus. The forest dynamics is acquired by maximizing the discounted present value of the revenue flow less the conversion and labor cost flows for given spontaneous regeneration.
Since LDCs have shown a decreasing stock of forest and an increasing stock of debt, only the initial conditions implying these dynamics have been considered.

It turns out that the stock of debt increases infinitely without repudiation under very general circumstances. Moreover, the stock of forest is likely to become completely depleted if its initial level is very small, and to oscillate around an equilibrium level otherwise.

These results have served as a benchmark against which outcomes originating from the DFNS and the DFDS are compared.

As regards the DFNSs, it turns out that the fraction of the external debt converted through these schemes must be close to the interest rate and the reduction in the deforestation rate must be large in order to ensure the inversion of the debt and forest dynamics, respectively. Moreover, the DFNSs may lead to the complete depletion of the stock of forest when its initial level is quite small. Finally, they are enforceable only when the stock of forest is small.

Hence, the analysis leads to the conclusion that the DFNS is effective, but enforceable only for countries characterized by the small stocks of forest.

As regards DFDSs, it appears that the proportion of external debt necessary to ensure the inversion of the debt dynamics is larger than is required for the DFNSs and that in the long run they reduce the forest’s equilibrium, apart from in countries characterized by an extremely small stock of forest, where they prevent its complete depletion.

Hence, the analysis leads us to conclude that the DFDS is enforceable, but effective only for countries characterized by an extremely small stock of forest.

Therefore the conclusion that can be drawn from the present analysis is that the debt-for-nature and the debt-for-development swaps can be combined when the LDC shows small stocks of forest and debt, whereas only the debt-for-nature swaps should be adopted when the LDC is characterized by a small stock of forest and a large stock of debt, and only the debt-for-development swaps should be agreed upon when the LDC shows a stock of forest that is not small and a small stock of debt.

References


Murphy, P. (1994), ‘Debt and deforestation in developing countries’, Department of Economics, University of California, Santa Barbara, CA.


Appendix 1
The relevant Hamiltonian to obtain the cleared land dynamics is given by:

\[
Ham_C = P_X^a L_X^b C^b - W L_X - \frac{1}{2} \rho H^2 + m_C \left[-(1 - C) + (1 - C)^\delta + H\right]
\]

The necessary conditions are given by:

\[
\frac{\partial Ham_C}{\partial L_X} = 0 \Leftrightarrow L_X = \left(\frac{\alpha P_X}{W}\right)^{1/(1-\alpha)} C^{\delta/(1-\alpha)}
\]

\[
\frac{\partial Ham_C}{\partial H} = 0 \Leftrightarrow H = \frac{m_C}{\rho}
\]

Therefore, cleared land and its shadow price must satisfy:

\[
\dot{C} = -(1 - C) + (1 - C)^\delta + \frac{m_C}{\rho}
\]

\[
m_c = -\frac{\partial Ham_C}{\partial C} = -m_c [1 - \delta (1 - C)^{\delta-1}] - \beta P_X^1/(1-\alpha) \left(\frac{\alpha}{W}\right)^{\alpha/(1-\alpha)} C^{(\alpha + \beta - 1)/(1-\alpha)}
\]

\[
\lim_{t \to \infty} m_C e^{-\alpha t} \geq 0
\]

\[
\lim_{t \to \infty} m_C e^{-\alpha t} = 0
\]

Appendix 2
The relevant Hamiltonian to obtain the debt dynamics is given by:

\[
Ham_D = (X - E_X)^A (I - P_X X + M_Y)^B - m_D (iD + M_Y - RE_X)
\]

The necessary conditions are given by:

\[
\frac{\partial Ham_D}{\partial X} = 0 \Leftrightarrow P_X = R
\]

\[
\frac{\partial Ham_D}{\partial E_X} = 0 \Leftrightarrow E_X = X - \frac{A}{RB} \left(\frac{B(A/(RB))^A}{m_D}\right)^{1/(1-A-B)}
\]

\[
\frac{\partial Ham_D}{\partial M_Y} = 0 \Leftrightarrow M_Y = P_X X - I + \left(\frac{B(A/(RB))^A}{m_D}\right)^{1/(1-A-B)}
\]

To simplify notation let \(R = A/B\). Therefore debt and its shadow price must satisfy:

\[
\dot{D} = iD - I + \left(1 + \frac{A}{B}\right) \left(\frac{B}{m_D}\right)^{1/(1-A-B)}
\]

\[
m_D = -\frac{\partial Ham_D}{\partial D} = im_D
\]

\[
\lim_{t \to \infty} m_D e^{-\alpha t} \geq 0
\]

\[
\lim_{t \to \infty} m_D e^{-\alpha t} = 0
\]
Appendix 3

The singularity set is defined as:

\[ S = \{(m_C, C, \alpha, \beta, A, B, \delta, \rho) \in \mathbb{R}^2 \times \mathbb{R}^6 \mid \text{Det}(\text{Hessian}) = 0\} \]

This is given by: \( I_c = I_{mc} \) and \( I_c' = I_{mc}' \) or, alternatively:

\[ \rho = \rho^*(\alpha, \beta, A, B, \delta, m_{c'}, C) = -\frac{I_{mc}}{(1 - C) - (1 - C)^\delta} \]

\[ \rho = \rho^{**}(\alpha, \beta, A, B, \delta, m_{c'}, C) = -\frac{I_{mc}'}{1 - \delta(1 - C)^\delta^{-1}} \]

Notice that:

\[ \frac{\rho^{**}}{\rho^*} = \frac{I_{mc}'}{I_{mc}} \frac{I_c'}{I_c} \]

The bifurcation set is defined as:

\[ Q = \{(\alpha, \beta, A, B, \delta, \rho) \in \mathbb{R}^6 \mid \text{Det}(\text{Hessian}) = 0 \text{ for } m_C = m_C(0), C = C(0)\} \]

This is given by: \( I_c = I_{mc} \) for \( m_C = m_C(0), C = C(0) \) and \( I_c' = I_{mc}' \) for \( m_C = m_C(0), C = C(0) \) or, alternatively:

\[ \rho = \rho^*(\alpha, \beta, A, B, \delta, m_{c}(0), C(0)) = -\frac{I_{mc}(0)}{(1 - C(0)) - (1 - C(0))^\delta} \]

\[ \rho = \rho^{**}(\alpha, \beta, A, B, \delta, m_{c}(0), C(0)) = -\frac{I_{mc}'(0)}{1 - \delta(1 - C(0))^{\delta^{-1}}} \]

It is easy to check that \( I_{mc}' < 0 \) for \( 0 < C < \hat{C} \) and \( I_{mc}' > 0 \) for \( \hat{C} < C < 1 - \delta^{1/(1-\delta)} \) and \( I_{mc}'' > 0 \). Moreover, \( I_c' > 0 \) and \( I_{mc}'' < 0 \) for \( 0 < C < 1 - \delta^{1/(1-\delta)} \). Consequently, \( (\rho^*') < 0 \) for \( 0 < C < \hat{C} \) and \( (\rho^*)' > 0 \) for \( \hat{C} < C < 1 - \delta^{1/(1-\delta)} \). Moreover, \( (\rho^{**})' > 0 \) for \( 0 < C < 1 - \delta^{1/(1-\delta)} \). Finally, notice that \( (\rho^*)' = 0 \Leftrightarrow \rho^* = \rho^{**} \).

Thus, no equilibrium exists for \( \rho < \rho^* \).

For \( \rho = \rho^* = \rho^{**} \) a bifurcation occurs at \( (m_C(0), C(0)) \), such that for \( \rho > \rho^* \) two equilibria appear: \( E^+ \) and \( E^- \). It will be proved that \( E^+ \) is a saddle, \( E^- \) is a centre.

A sufficient condition for an equilibrium to be a saddle point is that the eigenvalues of the Jacobian matrix are real and distinct, whereas a necessary condition for an equilibrium to be a centre is that the eigenvalues of the Jacobian matrix are complex with null real part. The couple \((m_{c'}, C)\) such that the eigenvalues are both null is given by:

\[ m_C = \frac{\rho[1 - \delta(1 - C^{\delta^{-1}})]}{\delta(\delta - 1)(1 - C)\delta^{-2}} + \beta[(A/B)C(\alpha + \beta - 1)]^{1/(1-\alpha)} \frac{(1 - \alpha)Z^\alpha}{(1 - \alpha)Z^\alpha \delta(\delta - 1)(1 - C)\delta^{-2}} \times \left[ \left( \frac{\alpha \beta}{Z} \right)^{1/(1-\alpha)} \left( \frac{A}{B} \right)^{C(\alpha + \beta - 1)} - \frac{(\alpha + \beta - 1)}{C} \right] \]

where: \( Z = 1 + (A/B)^{1/(1-\alpha)}C^B/(1-\alpha) \).
Let us call $I_e$ the right-hand side of this expression. Define $Hull(I_e) \equiv \{(m_C, C) \mid m_C > I_e, 0 < C < 1\}$. Therefore if $(m_C, C) \in Hull(I_e)$, eigenvalues are complex with null real part; otherwise eigenvalues are real and distinct. Notice that: $C = I_m$ and $C' = I_{mc}'$ imply $I_e = I_e$. Moreover $I_e = I_{mc}$ and $I_e' < I_{mc}'$ imply $I_e' < I_{mc}'$. Thus: $E' \notin Hull(I_e)$ and $E' \in Hull(I_e)$. Consequently, one can state that $E'$ is a saddle point. However:

$$\frac{\partial C}{C} + \frac{\partial m_c}{m_c} = 0$$

Consequentially, by Lie derivative (Lorenz, 1993) one can state that except for $E'$, every trajectory of equations (1) and (2) in $Hull(I_e)$ is a closed orbit. Moreover, by Green’s theorem (Hirsch and Smale, 1974), no limit cycle exists. Notice that this analysis is consistent with decreasing, constant, and increasing returns to scale.

**Appendix 4**
The invariant curve is increasing and concave in $t$, respectively, if and only if:

$$\frac{i \partial I_d}{\partial t} = \frac{(A + B)i[(B \exp[-it]) / m_D(0)]^{1/(1 - A - B)} - B(1 - A - B)}{B(1 - A - B)^2} + \frac{\beta z C'}{Z^a C} > 0$$

$$- i \frac{\partial^2 I_d}{\partial t^2} = \frac{(A + B)i^2[(B \exp[-it]) / m_D(0)]^{1/(1 - A - B)} + \beta(1 - \alpha)Z^{1 - \alpha}z[\alpha Zz(C')^2 + (1 - \alpha - \beta)(C')^2 - CC']}{(1 - \alpha)ZC^2} < 0$$

where: $Z = 1 + (A/B)^{1/(1 - \alpha)}C^{B/(1 - \alpha)}$ and $z = [(A/B)C^{\beta}]^{1/(1 - \alpha)}$. The invariant curve is increasing and concave in $C$, respectively, if and only if:

$$\frac{\partial I_d}{\partial C} = \frac{\beta(A/B)^{1/(1 - \alpha)}C^{-1 + \beta/(1 - \alpha)}}{i[1 + (A/B)^{1/(1 + \alpha)}C^{B/(1 - \alpha)}]^\alpha} > 0$$

$$\frac{\partial^2 I_d}{\partial C^2} < 0 \Leftrightarrow \alpha + \beta < 1$$

However, from the previous analysis one sees that $C$ can always decrease, oscillate (increase and decrease or the other way round), or always increase. Let us consider an $I_d$ concave in $C$ only. If $C$ decreases, then $D$ will always increase if $D(0) > I_d(0)$. If $C$ increases, then $D$ could increase and then decrease if $D(0) > I_d(0)$ provided the dynamics of $D$ is faster than that of $C$.

**Appendix 5**
The foreign debt will not be repudiated at time $t_r$ if the following condition holds:

$$\int_{I_r}^{\infty} [Z^{(-\alpha)}(Z - z) + M_r]^{A+B} e^{-\alpha t} dt < \int_{I_r}^{\infty} \left( \frac{B \exp[-it]}{m_D(0)} \right)^{(A+B)/(1 - A - B)} e^{-\alpha t} dt$$

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If for the sake of simplicity we assume $E_x = M_y = 0$, this condition boils down to:

$$\frac{1}{s} Z^{(-o)}[Z - z]$$

\[
\left( \frac{B}{m_D(0)} \right)^{1/(1-A-B)} \frac{1 - A - B}{i + \sigma(1 - A - B)} \exp\left[ \frac{-i + \sigma(1 - A - B)}{\sigma(1 - A - B)} t_r \right]
\]

This condition is never met if \( [i + \sigma(1 - A - B)]/(1 - A - B) < 0 \). If either \( 1 - A - B > 0 \) and \( i + \sigma(1 - A - B) > 0 \) or, alternatively, \( 1 - A - B < 0 \) and \( i + \sigma(1 - A - B) < 0 \), it can be solved with respect to \( t_r \).